

8.4 Solving Log & Exponential Equations

Solving Log Equations

Combine all log terms if possible.

Write in exponential form if helpful.

Answer in exact form if possible.

$$\text{ex) } \rightarrow \log_6(2x-1) = \log_6 11$$

$$2x-1 = 11$$

$$2x = 12$$

$$x = 6$$

VERIFY

$$\text{ex) } \log(8x+4) = 1 + \log(x+1)$$

$$\log(8x+4) = \log 10 + \log(x+1)$$

$$\log(8x+4) = \log(10(x+1))$$

$$8x+4 = 10x+10$$

$$-6 = 2x$$

$$-3 = x \quad \underline{\text{Verify}}$$

-3 is not a solution

$$\text{ex) } \log_2(x+3)^2 = 4$$

$$\log_2(\quad) = 4$$

$$\frac{2 \log_2(x+3)}{2} = \frac{4}{2}$$

$$\log_2(x+3) = 2$$

$$2^2 = x+3$$

$$4 = x+3$$

$$1 = x$$

$$\log_2(x+3)^2 = \log_2(16)$$

$$(x+3)^2 = 16$$

$$x+3 = \pm 4$$

$$x = 1, -7$$

$$y = \log_2(\dots(x+3)^2 \dots)$$

$$\begin{aligned} \text{ex) } \log_3(x^2 - 8x)^5 &= 10 \rightarrow \frac{5}{5} \log_3(x^2 - 8x) = \frac{10}{5} \\ (x^2 - 8x)^5 &= 3^{10} & \log_3(x^2 - 8x) &= 2 \\ x^2 - 8x &= 9 & 3^2 &= x^2 - 8x \\ x^2 - 8x - 9 &= 0 \leftarrow \\ (x - 9)(x + 1) &= 0 \\ \checkmark x = 9 \quad \checkmark x = -1 & \text{ verify} \end{aligned}$$

Solving Exponential Equations

Isolate the "power" first.

$$\boxed{5^x} \leftarrow \text{power}$$

Take the log of both sides (choose a convenient base).

ⓐ Write in log form.

Answer in exact form, unless otherwise stated.

$$\text{ex) } 4^x = 605 \rightarrow \log_4 605 = x$$

$$\log_4 4^x = \log_4 605$$

$$\boxed{x = \log_4 605}$$

$$x = 4.62\dots$$

$$\log_{10} 4^x = \log_{10} 605$$

$$\frac{x \log_{10} 4}{\log_{10} 4} = \frac{\log_{10} 605}{\log_{10} 4}$$

$$= \frac{\ln 605}{\ln 4}$$

$$\text{ex) } \frac{8(3^{2x})}{8} = \frac{568}{8}$$

$$(3^2)^x \leftarrow 3^{2x} = 71$$

$$9^x \quad \frac{\log_3 71}{2} = \frac{2x}{2}$$

$$\frac{1}{2} \log_3 71 = x$$

$$\text{ex) } 4^{2x-1} = 3^{x+2}$$

$$\log 4^{2x-1} = \log 3^{x+2}$$

$$(2x-1)\log 4 = (x+2)\log 3$$

$$2x\log 4 - \log 4 = x\log 3 + 2\log 3$$

$$-x\log 3 + \log 4 \quad -x\log 3 + \log 4$$

$$2x\log 4 - x\log 3 = 2\log 3 + \log 4$$

$$x(2\log 4 - \log 3) = 2\log 3 + \log 4$$

$$\frac{x(\log(16/3))}{\log(16/3)} = \frac{\log 36}{\log(16/3)}$$

$$x = \frac{\log 36}{\log(16/3)} = \log_{16/3} 36 \doteq 2.14\dots$$

verify using sub or calc.

pg. 412-413

1-8, 18, 21, 22, C1, C4